Pavel V. Shevchenko
Quantitative Risk Management

CSIRO
Mathematical & Information Sciences

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# Bridging to Finance

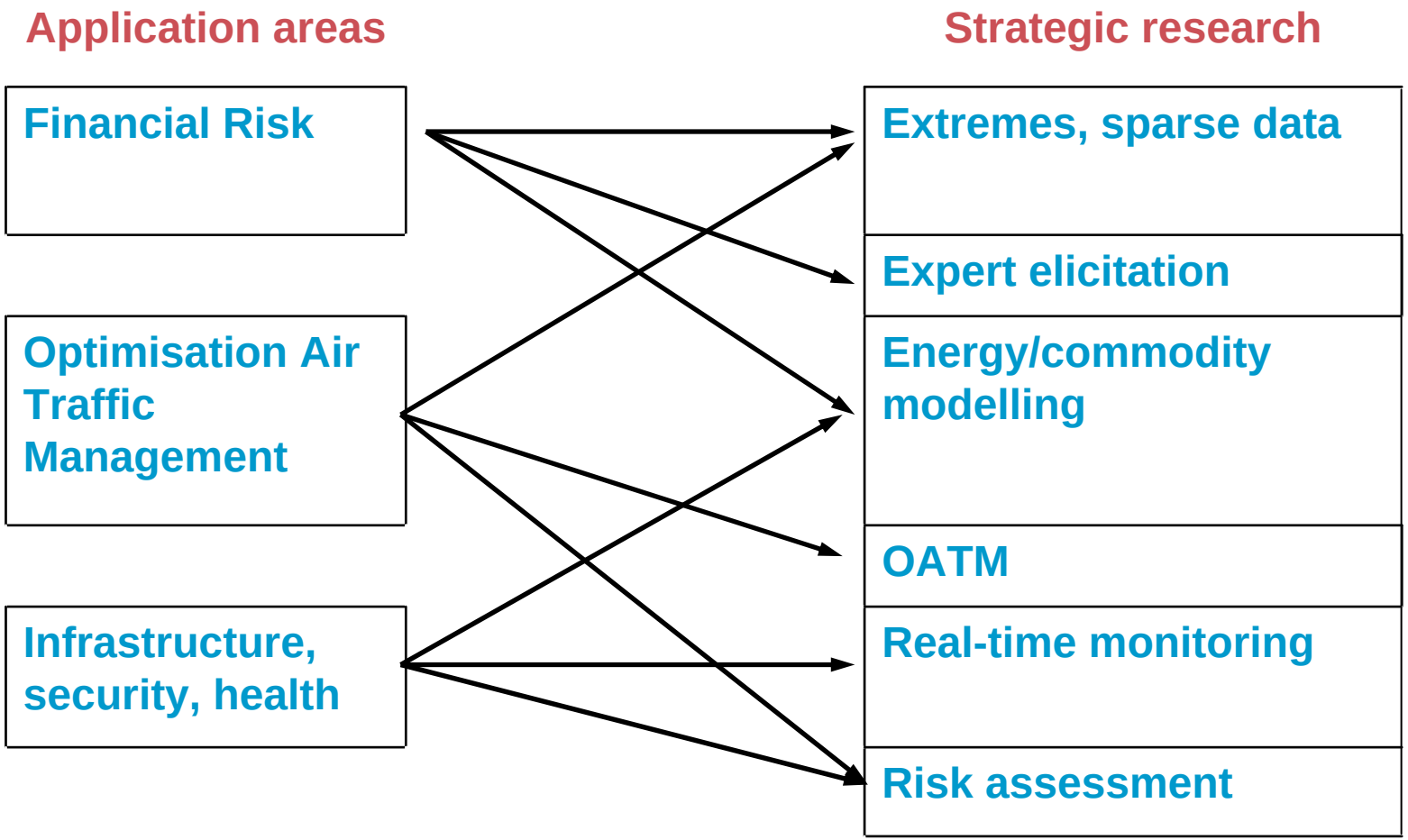
**Conference "*Quantitative Methods in Investment and Risk Management: sourcing new approaches from mathematical theory and the real world*"**
**Melbourne Centre For Financial Studies, 20th September 2007.**

CSIRO

# CSIRO Mathematical & Information Sciences

- **Commonwealth Scientific and Industrial Research Organization of Australia (CSIRO)**
  - National research agency formed in 1926.
  - Approx 6500 staff (Divisions: Industrial Physics, Minerals, Mathematical & Information Sciences, Marine and Atmospheric Research, etc.)
    www.csiro.au
- **Division of Mathematical and Information Sciences (CMIS)**
  - (over 100 researchers): Decision Technology, Biotechnology and Health Informatics, Environmental Informatics
    www.cmis.csiro.au
- **Quantitative Risk Management (QRM) group**
  - (approx. 20 staff): financial risk, infrastructure, environment risk, security, air-transport. Activities/modes of engagement: research, consulting, model development/validation, software development,… .
    www.cmis.csiro.au/QRM

# CSIRO Quantitative Risk Management



# CSIRO Mathematical & Information Sciences

## Solutions

- Development of mathematical models and customised software according to the client methodology and specific needs.
- Assisting with development of new models and their implementation into software.
- Independent review and advice on risk models, methodology and software solutions.
- Independent validation of derivatives and risk measurement models.
- Time-series analysis of data.

## Modes of engagement

There are many ways CSIRO can work with you to understand and quantify financial risks:

- Consultancy engagement.
- Contract research engagement.
- Collaborative projects.
- Software development.

# Track Records in Financial Risk

- **Derivative pricing:** work on CSIRO option pricing software Reditus since 1999, consulting projects in 2005, 2006, 2007.
- **FX option pricing:** plug-ins for Fenics launched in 2005 – 100 users in overseas banks.
- **Operational Risk:** validation projects in 2000, model development projects in 2001, 2003, R&D/software projects 2004-2007.
- **Market Risk:** validation projects in 2004, 2005, model development consulting project in 2007.
- **Credit Risk:** validation projects in 1999, 2002; validation and model development projects 2004-ongoing.
- **Underwriting risk:** consulting projects in 1999, current proposals.
- **Forecasting electricity/commodities:** consulting projects in 2002, 2005, 2005-06, current proposals.
- **Portfolio Management:** model development projects in 2007, current proposals.
- **Water/Carbon Trading:** current proposals.
- **Collaborators:** Monash Uni, Cambridge Uni, ETH Zurich, UNSW, UTS, Macquarie Uni, Statistical Research Associates NZ
- **Industry clients**: CBA, ANZ, NAB, St George, Integral Energy, IAG, Fenics FX, Edgecap, Moore Capital, Ester Bank, Credit Swiss,… .

## Financial Risk Management

- Market Risk
- Credit Risk
- Operational Risk
- Underwriting risk
- Derivative pricing
- Interest Rates
- Trading strategies
- Portfolio Management
- Commodity/Energy
- Carbon/water trading
- Model risk
- Liquidity risk

## Links

- Extreme Value modelling
- Dynamic control
- Expert Elicitation
- Bayesian methods
- Dependence Modelling
- Model validation
- Computational methods (PDE, MCMC, Monte Carlo methods)
- Time series analysis
- High performance computing

## Other Risk Areas

- Air transport
- Ecology/Environmental
- Infrastructure
- Security
- Weather/Climate
- Health

# Extreme Value Analysis/Dependence

- **Air-transport (deviations from taxi centerline)**
- **Weather (rainfall, wind speed)**
- **Market Risk (tail of portfolio return distribution, derivatives)**
- **Credit/Operational Risk (tail of annual loss distribution)**
- **Electricity pricing (price spikes)**

# Extreme Value Models

- **Block maxima – models for largest observations,** $M_n = \max(X_1,..., X_n)$

$X_1, X_2,...$ are iid **e.g. daily data grouped into quarterly blocks**

**Limiting distribution: Generalized Extreme Value Distribution (GEV)**

$$H(x) = \begin{cases} \exp[-(1+\xi \times x/\beta)^{-1/\xi}], \xi \neq 0 \\ \exp[-\exp(-x/\beta)], \quad \xi = 0 \end{cases}$$

- **Threshold exceedances – models for large observations exceeding some high level, *L*,** $Y_i = X_i - L$

***e.g. operational losses exceeding 1mln***

**Limiting distribution: Generalized Pareto Distribution (GPD)**

$$H(x) = \begin{cases} 1-(1+\xi x/\beta)^{-1/\xi}; & \xi \neq 0 \\ 1-\exp[-x/\beta]; & \xi = 0 \end{cases}$$

# Market Risk

$$X_t = -(\ln S_t - \ln S_{t-1}) \approx (S_{t-1} - S_t)/S_{t-1}$$

$$X_t = \mu_t + \sigma_t Z_t$$

assume $Z_t, Z_{t-1}, \ldots$ are iid and $X_t$ is stationary

For example : $\mu_t = \lambda X_{t-1}, \sigma_t^2 = \alpha_0 + \alpha_1 (X_{t-1} - \mu_{t-1})^2 + \beta \sigma_{t-1}^2$

$$VaR_q^t = \mu_{t+1} + \sigma_{t+1} VaR(Z)_q; CVaR_q^t = \mu_{t+1} + \sigma_{t+1} CVaR(Z)_q$$

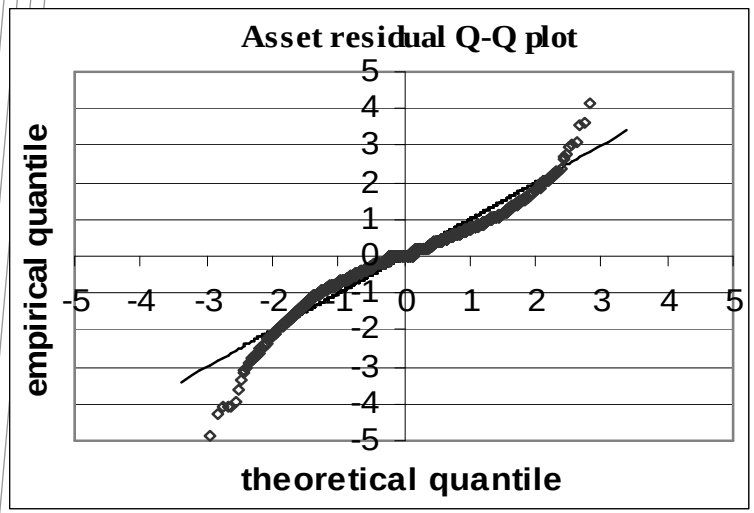
$Z$ is modelled by $GPD$

Market Risk Regulatory Capital

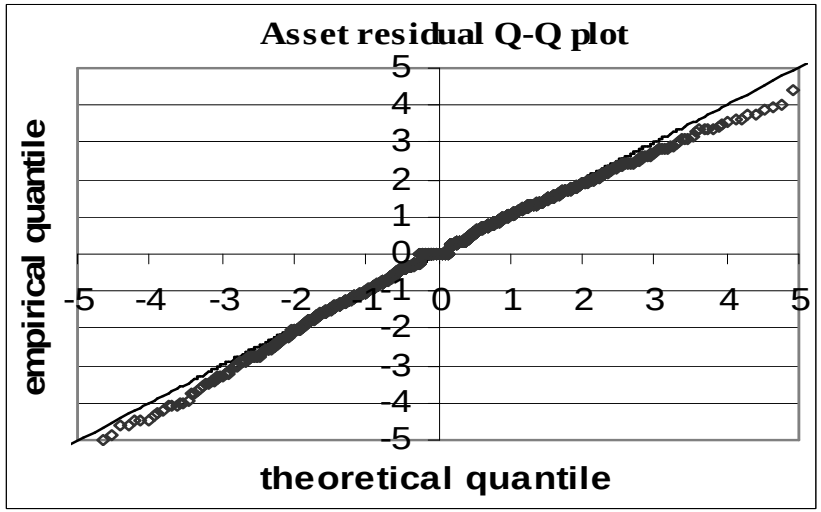
$$C(t) = \max\{\mathrm{VaR}_{0.99}^{t,10}, \frac{k}{60} \sum_{i=1}^{60} VaR_{0.99}^{t-i+1,10}\}$$

# Example: API returns

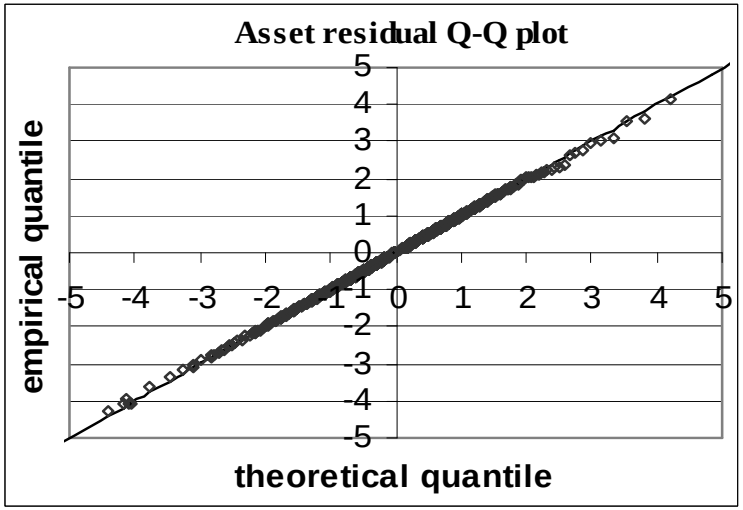
## GARCH-Normal



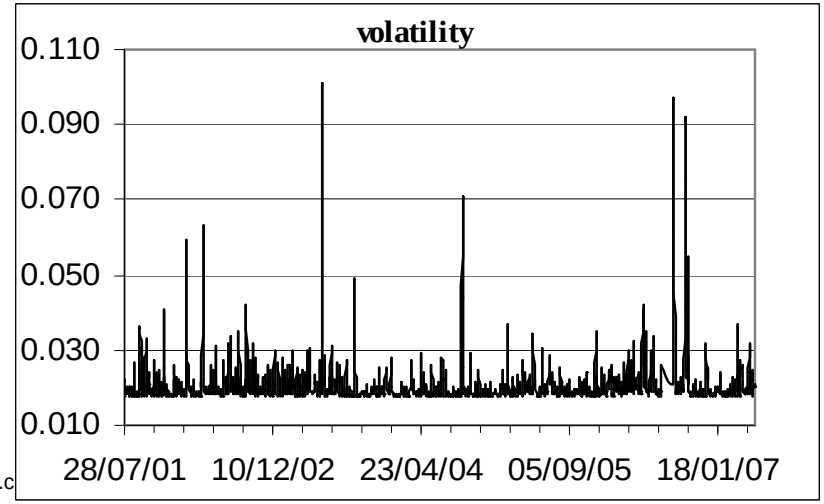

## GARCH-Student t




## GARCH-Generalized Pareto




## GARCH-volatility



# Derivative Pricing

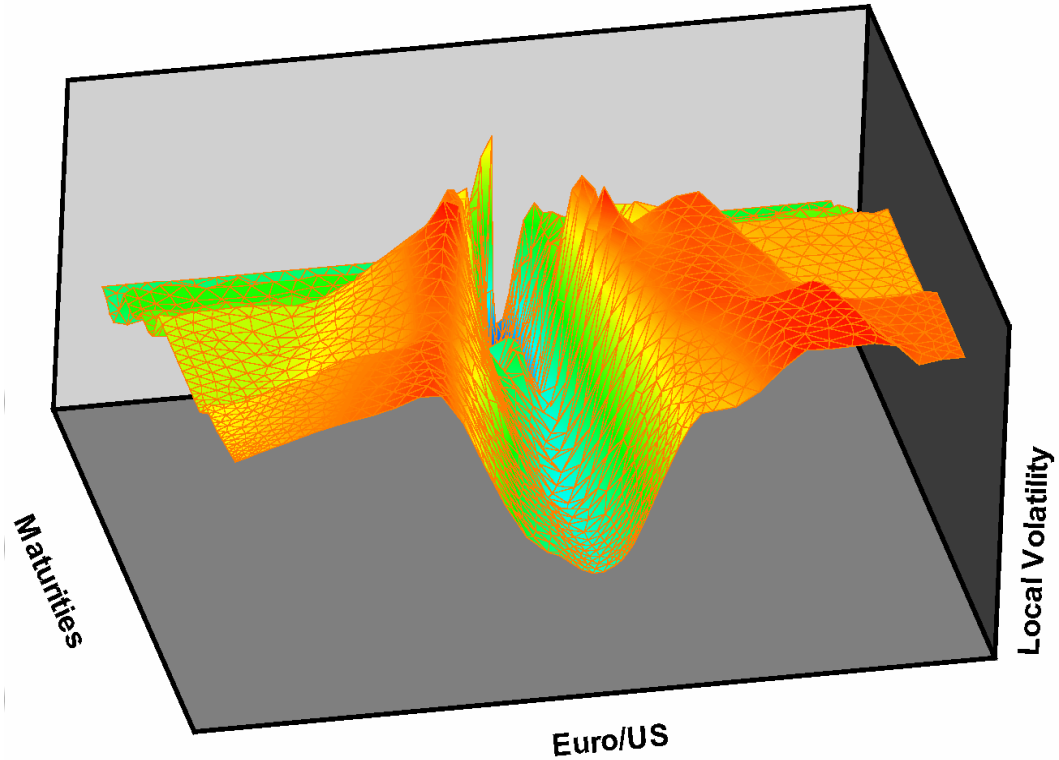
Option pricing : $Q_t = E[Payoff(S_T)|S_t]$

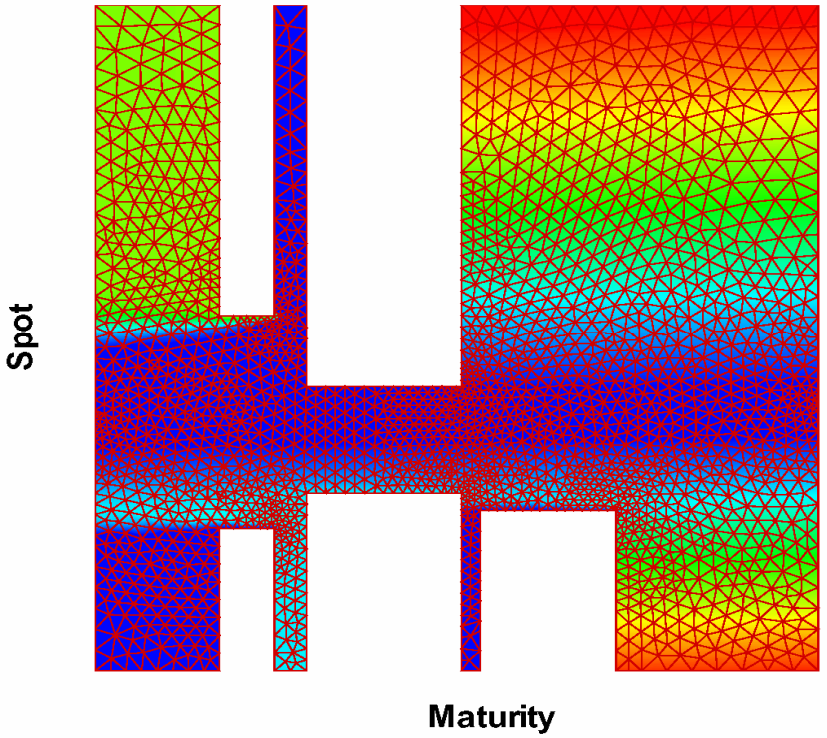
Modelling volatility skew :

e.g. local volatility models - $dS_t / S_t = (r_t - q_t)dt + \sigma(S_t, t)dW_t$,

stochastic volatility models, jump diffusion

Numerical Methods : Finite Element, Finite Difference, Monte Carlo methods.



# Dependence modelling

## Dependence modelling via Copula method

Consider rvs $X_1,\ldots,X_d$ with $F_i(x_i)=\text{Prob}(X_i \le x_i)$

$$F_i^{-1}(U_i)\sim F_i \Rightarrow X_i = F_i^{-1}(U_i),\ U_i \sim U(0,1),\ F_i(X_i)=U_i \sim U(0,1)$$

$F(x_1,x_2,\ldots,x_d)=\text{Prob}[X_1\le x_1, X_2 \le x_2,\ldots,X_d \le x_d]$ is joint cdf

$$F(x_1,x_2,\ldots,x_d)$$

marginal behavior, $F_i$ copula $C(u_1,..,u_d)$

## Copula is multivariate joint distribution of uniform random variables

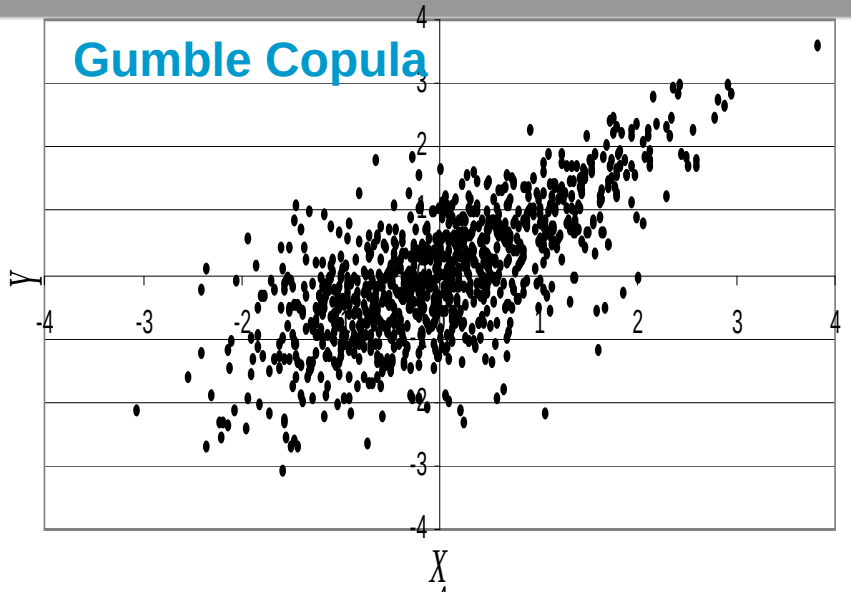
$$C(u_1,u_2,\ldots,u_d)=F(F_1^{-1}(u_1),F_2^{-1}(u_2),\ldots,F_d^{-1}(u_d))$$

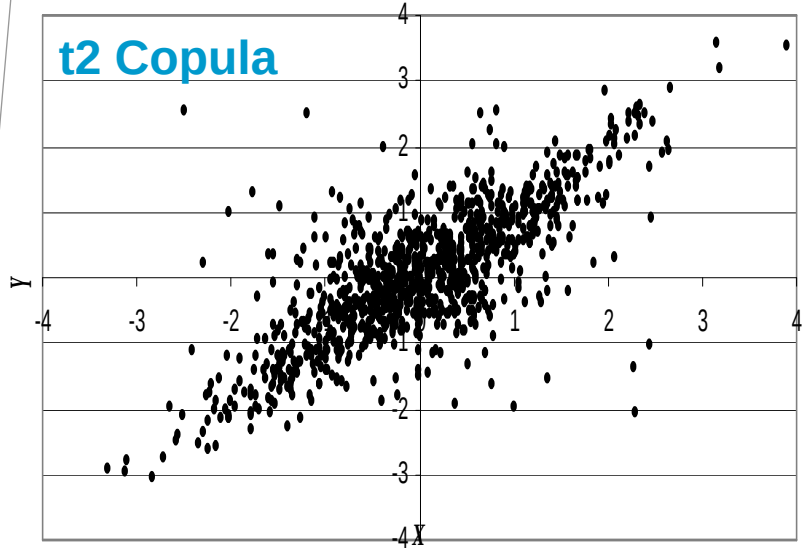
$$C(F_1(x_1),F_2(x_2),\ldots,F_d(x_d))=F(x_1,x_2,\ldots,x_d)$$

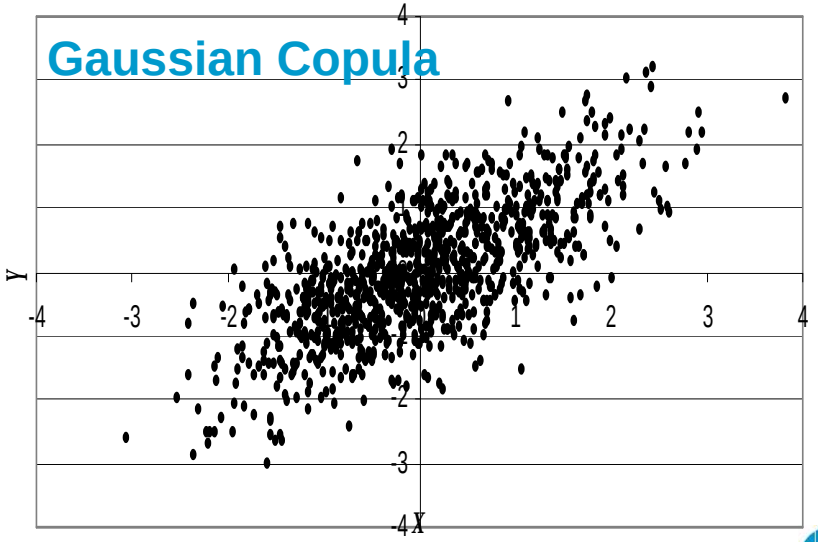
# Dependence modelling via copula

$$X \sim Normal(0,1); Y \sim Normal(0,1)$$

$$corr(X, Y) = 0.7$$



# Expert Elicitation/Bayesian methods

(combining internal & external data with expert opinion)

- **Ecology (estimation of fish density from gillnet catches)**
- **Operational Risk (estimation of loss frequency and severity)**
- **Air-transport (individual&collective risk of air-collisions)**
- **Insurance (pricing of policy premium)**
- **Markov Chain Monte Carlo methods (signal processing)**

- **Recent Publications:**
- **D. D. Lambrigger (ETH), P.V. Shevchenko (CSIRO) and M. V. Wüthrich (ETH),** 2007. *The Quantification of Operational Risk using Internal Data, Relevant External Data and Expert Opinions*. The Journal of Operational Risk.
- **Hans Bühlmann (ETH), P.V. Shevchenko (CSIRO) and M. Wüthrich (ETH),** 2006. A *"Toy" Model for Operational Risk Quantification using Credibility Theory.* The Journal of Operational Risk 2(1).
- **P.V. Shevchenko (CSIRO) and M. Wüthrich (ETH),** 2006. *Structural Modelling of Operational Risk using Bayesian Inference: combining loss data with expert opinions.* The Journal of Operational Risk 1(3), pp.3-26.

# Combining internal data, industry data and expert opinions

## Bayesian inference

**observations** $\mathbf{X} = (X_1, X_2, ..., X_n)$ **parameters** $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_K)$

$$h(\mathbf{X}, \boldsymbol{\theta}) = h(\mathbf{X} \mid \boldsymbol{\theta})\pi(\boldsymbol{\theta}) = \hat{\pi}(\boldsymbol{\theta} \mid \mathbf{X})h(\mathbf{X})$$

$$\hat{\pi}(\boldsymbol{\theta} \mid \mathbf{X}) \propto h(\mathbf{X} \mid \boldsymbol{\theta})\pi(\boldsymbol{\theta})$$

$\pi(\boldsymbol{\theta})$ **prior distribution is estimated by expert/industry data**

$h(\mathbf{X} \mid \boldsymbol{\theta})$ **likelihood of internal observations**

$\varphi(X_{n+1} \mid \mathbf{X}) = \int g(X_{n+1} \mid \boldsymbol{\theta}) \times \hat{\pi}(\boldsymbol{\theta} \mid \mathbf{X}) d\boldsymbol{\theta}$ **predictive distribution**

Combining internal data, external data and expert opinions

$$\hat{\pi}(\boldsymbol{\theta} \mid \mathbf{X}, \upsilon) \propto h_1(\mathbf{X} \mid \boldsymbol{\theta}) h_2(\upsilon \mid \boldsymbol{\theta}) \pi(\boldsymbol{\theta})$$

$\pi(\boldsymbol{\theta})$ **prior distribution is estimated by industry data**

$h_1(\mathbf{X} \mid \boldsymbol{\theta})$ **likelihood of internal observations**

$h_2(\upsilon \mid \boldsymbol{\theta})$ **likelihood of expert opinions**

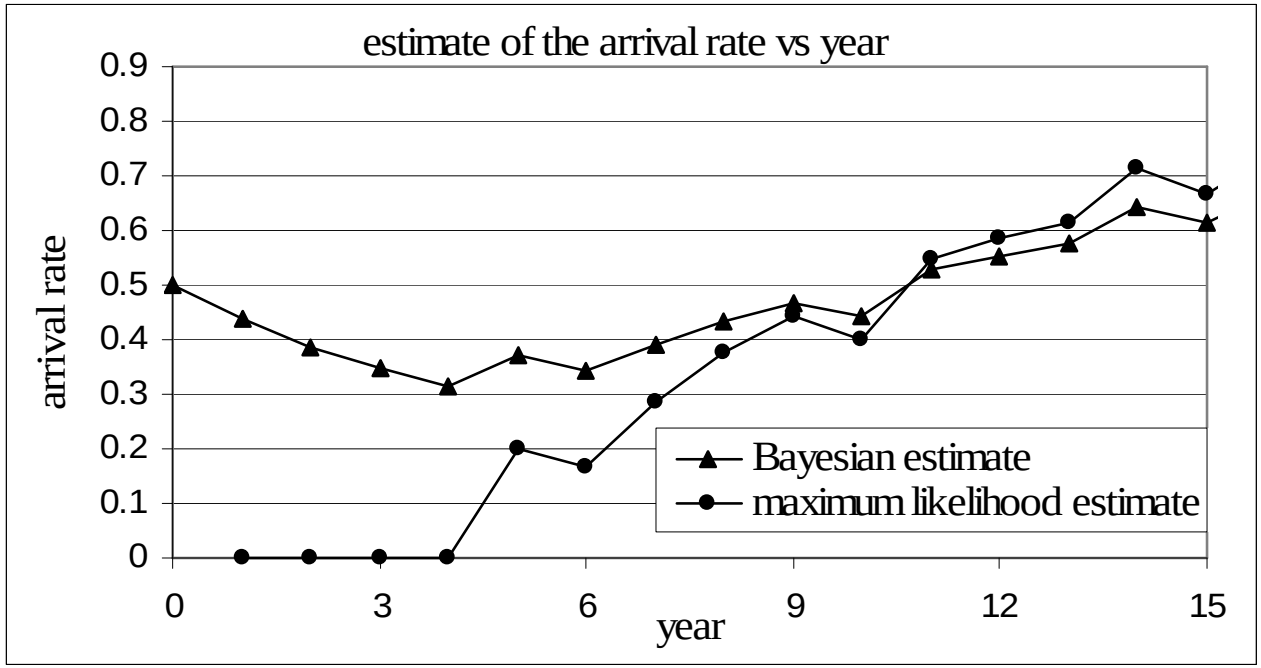
# Example: combining expert opinion and internal data

Annual counts **N**=(0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 2, 1, 1, 2, 0) from Poisson $\lambda = 0.6$

expert opinions $E[\lambda] = 0.5, \Pr[0.25 \le \lambda \le 0.75] = 2/3 \Rightarrow \alpha \approx 3.41, \beta \approx 0.15$

$\hat{\lambda}_k = \hat{\alpha}_k \times \hat{\beta}_k$ the Bayesian estimator with Gamma prior $\alpha \approx 3.41, \beta \approx 0.15$

$\tilde{\lambda}_k = \frac{1}{k}\sum_{i=1}^{k} N_i$ the Maximum Likelihood estimator



# Parameter Risk (uncertainty of parameters)

$$Z = \sum_{i=1}^{N} X_i - \text{annual loss}; \quad \mathbf{Y} = (\mathbf{X}, \mathbf{N}) - \text{past} \quad \text{observations}$$

$$\varphi(Z_{t+1} \mid \mathbf{Y}) = \int g(Z_{t+1} \mid \boldsymbol{\theta}) \times \hat{\pi}(\boldsymbol{\theta} \mid \mathbf{Y}) d\boldsymbol{\theta} - \text{predictive distribution}$$

$$\hat{Q}^{B}_{0.999} - 0.999 \text{ quantile of} \quad \varphi(Z_{t+1} \mid \mathbf{Y})$$

$$\hat{Q}_{0.999} - 0.999 \text{ quantile of} \quad g(Z_{t+1} \mid \hat{\boldsymbol{\theta}})$$

$\hat{\boldsymbol{\theta}}$ is point estimator, e.g. maximum likelihood estimator

$$\hat{\pi}(\boldsymbol{\theta} \mid \mathbf{Y}) \propto h(\mathbf{Y} \mid \boldsymbol{\theta}) \pi(\boldsymbol{\theta}), \quad \text{Markov Chain Monte Carlo methods}$$

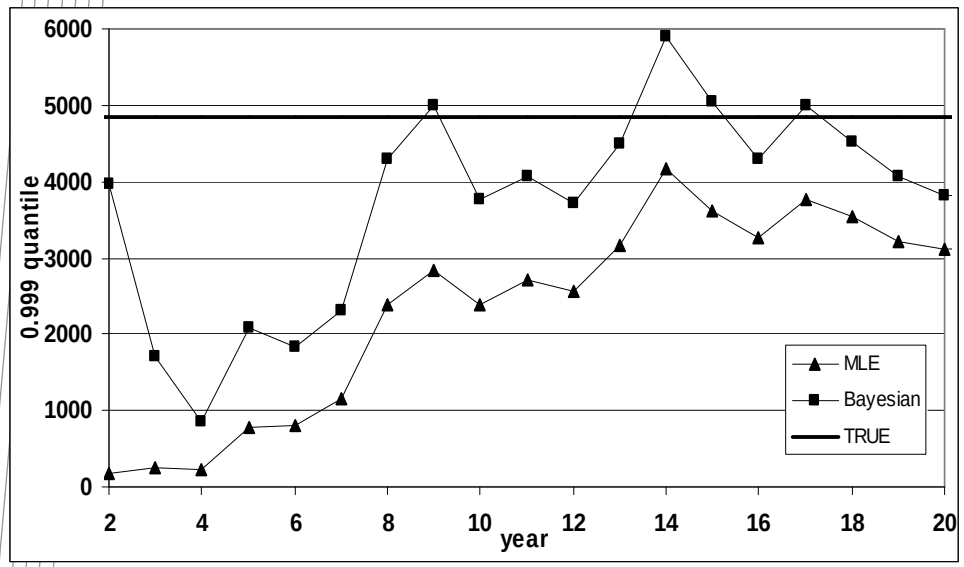
$$bias = E[\hat{Q}^{B}_{0.999} - \hat{Q}_{0.999}]$$

$$\text{Normal approximation}: \hat{\pi}(\hat{\boldsymbol{\theta}} \mid \mathbf{Y}) \text{ is } Normal(mean = \hat{\boldsymbol{\theta}}, \text{cov} = -\mathbf{I}^{-1})$$

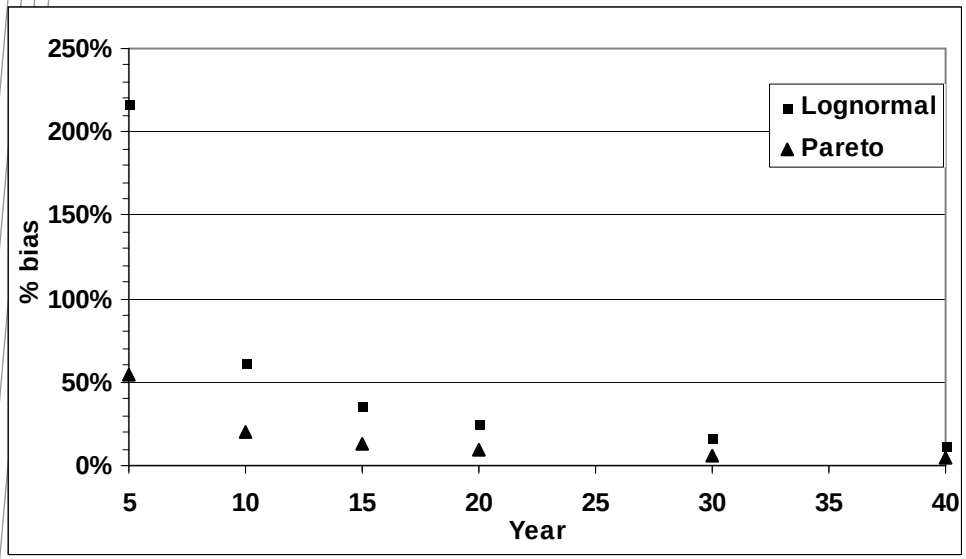
$$\ln \hat{\pi}(\boldsymbol{\theta} \mid \mathbf{Y}) \approx \ln \hat{\pi}(\hat{\boldsymbol{\theta}} \mid \mathbf{Y}) + \frac{1}{2} \sum_{i,j} \mathbf{I}_{ij} (\theta_i - \hat{\theta}_i)(\theta_j - \hat{\theta}_j); \quad \mathbf{I}_{ij} = \left. \frac{\partial^2 \ln \hat{\pi}(\boldsymbol{\theta} \mid \mathbf{Y})}{\partial \theta_i \partial \theta_j} \right|_{\theta = \hat{\theta}}$$

**P.V. Shevchenko (2007).** *Estimation of operational Risk Capital Charge under Parameter Uncertainty,* submitted to the Risk Magazine.

# Parameter Risk (uncertainty of parameters)




**Comparison of the 0.999 quantile estimators. Parameter uncertainty is ignored by (MLE) but is taken into account by (Bayesian). MLE - maximum likelihood estimator Bayesian - quantile of predictive distribution Losses were simulated from *Poisson*(10) and *LN*(1,2). Non-informative constant prior were used.**




**Relative bias in the 0.999 quantile estimator induced by the parameter uncertainty vs number of observation years. (Lognormal) - losses were simulated from *Poisson*(10) and *LN*(1,2). (Pareto) – losses were simulated from *Poisson*(10) and *Pareto*(2) with *L*=1.**

# Kalman/Particle filter techniques
## (state-space models)

- **On-line monitoring of water quality**
- **Health surveillance**
- **Modelling commodities/interest rates**

# State-space models: Kalman Filter

- **Measurement Equation** $\vec{F}_t = \vec{A} + \hat{B} \times \vec{X}_t + \vec{e}$
- **Transition Equation** $\vec{X}_{t+1} = \vec{M} + \hat{T} \times \vec{X}_t + \vec{\varepsilon}$

## Commodity spot models:e.g. 2-factor convenience yield

$S_t$ – spot price; $\delta_t$ – convenience yield, $F_{t,T}$ – futures prices

$$d \ln S_t = [\mu - \delta_t - \tfrac{1}{2}\sigma_1^2]dt + \sigma_1 dW_t^{(1)}$$

$$d\delta_t = [\alpha - \kappa_1 \delta_t - \kappa_2 X_t]dt + \sigma_2 dW_t^{(2)}; \quad E[dW_t^{(1)} dW_t^{(2)}] = \rho dt$$

$$F_{t,T} = E[S_T \mid S_t, \delta_t] \Rightarrow \ln F_{t,T} = A(T-t) + B(T-t)\ln S_t + C(T-t)\delta_t$$

## Interest rate spot models:e.g. Vasicek model

$r_t$ – short - term rate; $P_{t,T}$ – bond price

$$dr_t = \alpha[\gamma - r_t]dt + \sigma dW_t$$

$$P_{t,T} = A(t,T)\exp[-B(t,T)r_t] \Rightarrow \ln P_{t,T} = \ln A(T-t) - B(T-t)r_t$$

# Emerging over-arching research topics

- **Mixing internal & external data with expert opinions (credibility theory, Bayesian techniques)**
- **Dependence between risks: copula methods, structural models**
- **Compound point processes**
- **Modelling distribution tail: EVT, mixed distributions, splices**
- **Efficient Markov Chain Monte Carlo, Monte Carlo, finite element/finite difference methods**
- **State-space models (sequential Monte Carlo, Kalman filter)**
- **Time series analysis via chaos theory methods**
- **Modelling truncated/censored data**
- **Nonlinear optimization with constraints**

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# Thank you

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